

Reinventing the PMI: Scenarios Task

Reference:

Norton, A., Arnold, R., Kokushkin, V., & Tiraphatna, M. (2023). Addressing the cognitive gap in mathematical induction. *International Journal of Research in Undergraduate Mathematics Education*, 9(2), 295-321.

Suppose $P(n)$ is a statement about a positive integer n , and we want to prove:

$P(n)$ is true for all positive integers n .

Each part below provides given information that is known to be true. For each part, decide with your group whether this information is enough to prove $P(n)$ is true for all positive integers n . If the answer is yes, no justification is necessary. If the answer is no, explain why.

1. $P(1)$ is true; for all integers $k \geq 1$, $P(k)$ is true.
2. $P(1)$ is true; there is an integer $k \geq 1$ such that $P(k) \rightarrow P(k + 1)$.
3. $P(1)$ is true; for all integers $k \geq 1$, $P(k) \rightarrow P(k + 1)$.
4. For all integers $k \geq 1$, $P(k) \rightarrow P(k + 1)$.
5. $P(1)$ is true; for all integers $k \geq 2$, $P(k) \rightarrow P(k + 1)$.
6. $P(1)$ and $P(2)$ are true; for all integers $k \geq 2$, $[P(k - 1) \wedge P(k)] \rightarrow P(k + 1)$.