Using Euler Diagrams to Determine the Validity of an Implication

Let $P(x)$ and $Q(x)$ be open statements in some universe U.

(a) Given an implication $P \to Q$ that is **known to be true**, what are all the possible ways that an Euler diagram depicting the truth sets for P and Q might look?

(b) Given an implication $P \to Q$ that is **known to be false**, what are all the possible ways that an Euler diagram depicting the truth sets for P and Q might look?

Counterexample & Negation

- An element $x \in U$ is a **counterexample** to the implication $P \to Q$ provided:

- How might we denote the set of all counterexamples to $P \to Q$?

- When is the implication $P \to Q$ **false**?

- How is the truth of $P \to Q$ related to its set of counterexamples?

- What is the negation of the implication $P \to Q$?

If $P \to Q$ is true, then its **converse** $Q \to P$ need not be true.

Why?

The statement ***P* if and only if *Q*** is called an **equivalence** (or **biconditional statement**)

Notation:

Why is this useful? Can anyone think of a mathematical equivalence that they've seen before?

Theorem. The implication $P \to Q$ is logically equivalent to its **contrapositive** $\sim Q \to \sim P$.

- What can be said about the truth sets of two statements that are logically equivalent? What about the false sets?

- How might we argue that the above theorem must be true for all statements P and Q?

Example 2

Determine whether $P \to Q$ is logically equivalent to the disjunction $\sim P \vee Q$. Justify your answer.