

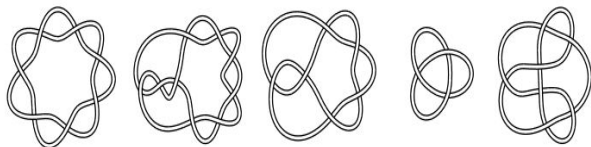
Torus Knots

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Introduction to Knot Theory

- Study of closed curves in 3 dimensional space
- As if a string was tied into a knot and the ends were joined [1]
- Knots differentiated by number of over crossings, undercrossings and if they can be untangled
- Knot theory is specifically interested in whether one knot can be transformed into another using Reidemeister moves
 - Reidemeister moves are the allowable manipulations of a knot that can be completed without altering the knot
 - Slides, twists, and pokes
 - Eliminates cutting, passing through, etc.
- C.F. Gauss (1777-1855) was the first known person to show interest in knot theory [2]

Figure 1 [7]



Torus Knots

- A (q,p) -torus knot is obtained by looping a string through the hole of a torus p times with q revolutions before joining its ends, where p and q are relatively prime. [3]
- Knots that do not include any points of intersection on the trivial torus
 - Torus knots can vary in size but all have a similar general shape
- Trivial torus is a torus formed by rotating the circle $m: (x-2)^2+y^2=1$ (centered at $(2,0)$, radius of 1) around the y -axis [2]
- Number of crossings can vary but all torus knots share similar donut shape

Applications

- Classification of atoms
- Modeling particle motion
 - Used when dealing with complex systems that involve torsion, curvature, or quantum mechanics
- Antenna design [4]
 - Trefoil knot (simplest torus knot)
 - Easy to manufacture through 3D printing
 - Stable radiation pattern

Figure 2 [4]

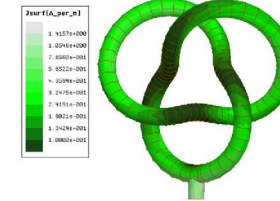


Figure 3 [5]

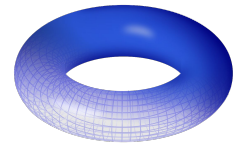
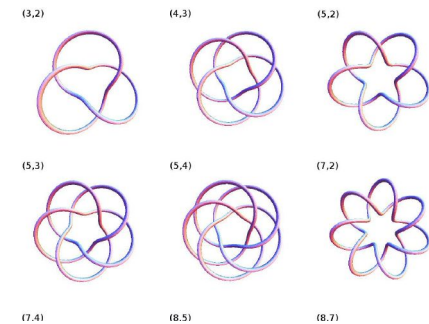


Figure 4 [6]



Further Research

- Do torus knots occur naturally?
- Are torus knots used in DNA?

References

- [1] Livingston, C. (2012). *Knot theory* (Ser. Carus, v. 24). Cambridge University Press. <https://ebookcentral.proquest.com/lib/vt/reader.action?docID=3330352>
- [2] Murasugi, Kunio (2008). *Knot Theory and Its Applications*. Birkhäuser <https://doi.org.ezproxy.lib.vt.edu/10.1007/978-0-8176-4719-3>
- [3] Weisstein, Eric W. "Torus Knot." From *MathWorld*—A Wolfram Web Resource. <https://mathworld.wolfram.com/TorusKnot.html>
- [4] Kumar, S. V., & Harish, A. R. (2016). Trefoil torus knot monopole antenna. *Ieee Antennas and Wireless Propagation Letters*, 15. <https://doi.org/10.1109/LAWP.2015.2453198>
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