Squaring the Circle

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Introduction

- Given: A unit circle
- Goal: Construct a square with the same area

Notice: We are attempting to construct a square with side length $\sqrt{\pi}$

Anaxagoras:
Greek philosopher was the first mathematician who attempted to square the circle in the 5th century BCE. [4]

[6]

James Gregory:
A Scottish mathematician applied the idea of sequences and convergence to prove there was no plane construction for squaring the circle. [4]

[7]

Impossible?

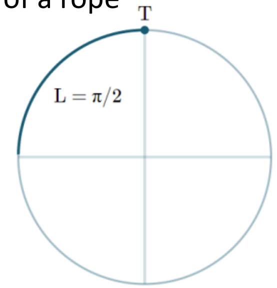
Lindemann-Weierstrass Theorem:

- π **is a transcendental number** and, therefore, **not a constructible number**. [5]
- Its property of being irrational and not being a root to polynomials with rational number coefficients shows that the $\sqrt{\pi}$ for the side length of a square to equal the area of a circle is not possible.

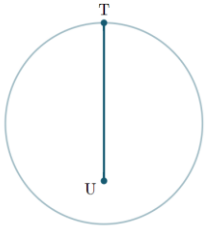
Proof: Rope Method

It is possible to square the circle with the addition of a rope

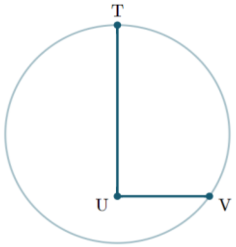
- Start with a circle with radius 1
- Measure $\frac{1}{4}$ of the circumference of the circle from the point T where T is a point on the circle.
- Since $C = 2\pi$, $\frac{1}{4}$ of 2π is $\frac{\pi}{2}$. So $\frac{\pi}{2}$ is the length of the quarter of our circle from point T.



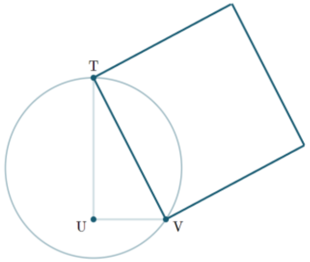
- Extend the "rope" of the L = $\frac{\pi}{2}$ along the diameter of the circle. (vertically) This creates a new segment TU.



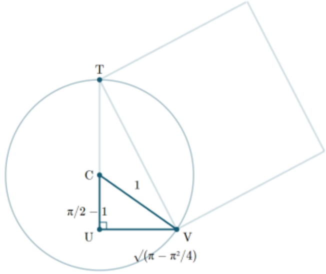
- Construct a horizontal line, from U to a point on the edge of the circle, V. This new line UV must be perpendicular to TU.



- Connect points V and T to create a right triangle.
- Then construct a square from the new segment VT. This segment VT = $\sqrt{\pi}$ is the side of a square with an area equal to the circle.



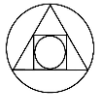
- In the right triangle CUV we have:
  CU = $\frac{\pi}{2}$-1 (since the rope length is $\frac{\pi}{2}$)
  CV = 1 (our circle has radius of 1).
  However, the $\sqrt{\pi}$ is technically impossible as stated previously



[2]

Interesting facts

- A square within a triangle within a larger circle began to be used in the 17th century to represent alchemy and the philosopher's stone.



- Often used in literature to denote the impossibility of something.
- Philosophically and spiritually, to understand the meaning of life, to be whole, complete, and free.

[1]

Further Questions

- Is it possible to Pentagon the Circle? Or any other shapes?
- What exactly are constructible numbers?

Citation

[1] Beyer, C. (n.d.). Squaring the circle is a geometric impossibility and an alchemy symbol. Retrieved April 29, 2021, from https://www.learnreligions.com/squaring-the-circle-96039

[2] Bourne, Murray. "Squaring the Circle Rope Method." *Intmathcom RSS*,Interactive Mathmatics, 2017, www.intmath.com/plane-analytic-geometry/squaring-the- circle.php.

[3]HH, Patriarch Sir Godfrey Gregg D.Div. "SQUARING THE CIRCLE." *THE MYSTICAL COURT*, THE MYSTICAL COURT, 14 Oct. 2017, mysticalcourt.com/2017/10/14/squaring-the-circle/.

[4] O'Connor, J J, and E F Robertson. "Squaring the Circle." *Maths History*, 1999, mathshistory.st-andrews.ac.uk/HistTopics/Squaring_the_circle/.

[5] Pierce, Rod. "Transcendental Numbers" Math Is Fun. Ed. Rod Pierce. 26 Nov 2020. 27 Apr 2021 <http://www.mathsisfun.com/numbers/transcendental-numbers.html

[6] "Anaxagoras - Greek Philosopher." *Crystalinks*, www.crystalinks.com/anaxagoras.html.

[7] David Stewart Erskine, 11th Earl of Buchan. "Professor James Gregory, 1638 - 1675. Mathematician." *National Galleries of Scotland*, National Galleries of Scotland, www.nationalgalleries.org/art-and-artists/2620/professor-james-gregory-1638-1675-mathematician