Nash Equilibrium

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Introduction

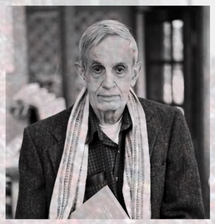
Nash equilibrium (NE) was a substantial contribution to economics and game theory.

The basic Game Theory model consists of a game with n players and n strategies. [1]

Nash equilibrium proposes that a set of strategies for each player would be the best overall outcome if each player had no incentive to stray from their own strategy. [1]

Background Info

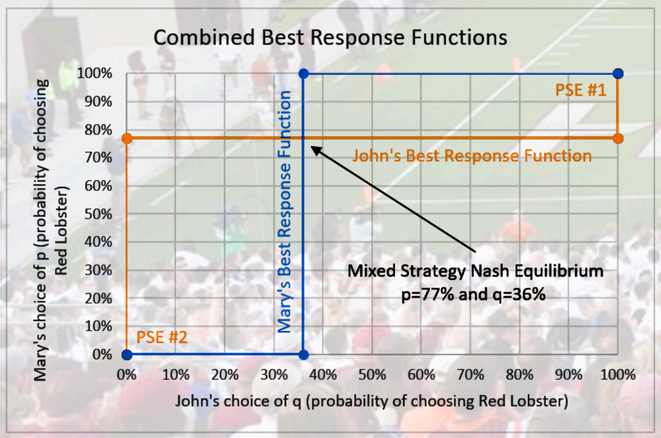
In 1950, mathematician and Princeton grad, John Forbes Nash created the idea of Nash equilibrium. [1]



Nash won the Nobel Prize in 1994 for his contribution to Economics and Game Theory. [4]

Nash's equilibrium helps us understand strategic behavior in economics, psychology, evolutionary biology and other fields. [2]

NE's influence on economic theory "is comparable to that of the discovery of the DNA double helix in in the biological sciences," said another Economics Nobel Prize winner, Roger Myerson. [2]



The Strategy

- n-person game in which each player has a set of strategies [5]

- Takes a few rounds for players to learn each others preferences [2]

- Strategy: calculate the expected payoff of the other player as a function of the probability distributions, then adjust theirs to "cancel out" the others

- no player can improve his outcome by changing only his own strategy [4]

- Nash equilibrium requires several conditions to hold in order to apply [3]

Assumptions

- All players are solely trying to increase their own perceived payoff, and will choose a strategy based on this

- All players preform their intended strategies perfectly

- All players smart enough to determine the solution

- All players can figure out the planned strategy of every other player, and understand that changing their surmised strategy will not cause another player to change their strategy

- All of the above is commonly understood, meaning that all players know each other fits the above assumptions [3]

Formal Definition

Let S_i denote the set of strategies for the i^{th} player, and $S = S_1 \cdot S_2 \cdot \ldots \cdot S_n$ denote the set of strategy profiles.

Let $f_i(s)$ denote the payoff to player i when evaluated at strategy profile $s \in S$

A Nash equilibrium is a strategy profile $s = (s_1, s_2, \ldots, s_n)$ with the property that

$$f_i(s) \geq f_i((s_1, s_2, \ldots, s_i', \ldots, s_n))$$

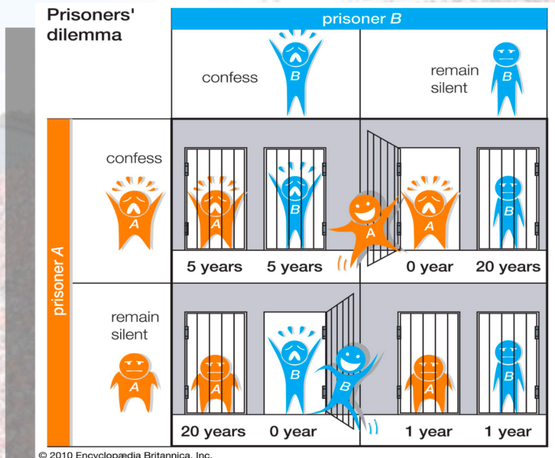
for all i, where $s_i' \in S_i$ denotes a strategy other than s_i available to player i. [3]

Examples & Applications

Coordination Game

Prisoner's Dilemma

The most well known example of NE is the prisoners dilemma, where two prisoner's time in jail is dependent not only on whether or not they confess, but also if the other confesses or not.



Further Questions

How has NE led to development of other equilibria and game theory ideas?

Beyond mathematics and economics, how has NE contributed to biological and social sciences?

References

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[3] Katz, Alexander, et al. "Nash Equilibrium." Brilliant Math amp; ScienceWiki, brilliant.org/wiki/nash-equilibrium/
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[5] Nash, John F. "Equilibrium Points in n-Person Games." Proceedings ofthe National Academy of Sciences of the United States of America, vol. 36,no. 1, 1950, pp. 48–49. JSTOR, www.jstor.org/stable/88031. Accessed13 Apr. 2021.