Markov Chains

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Introduction

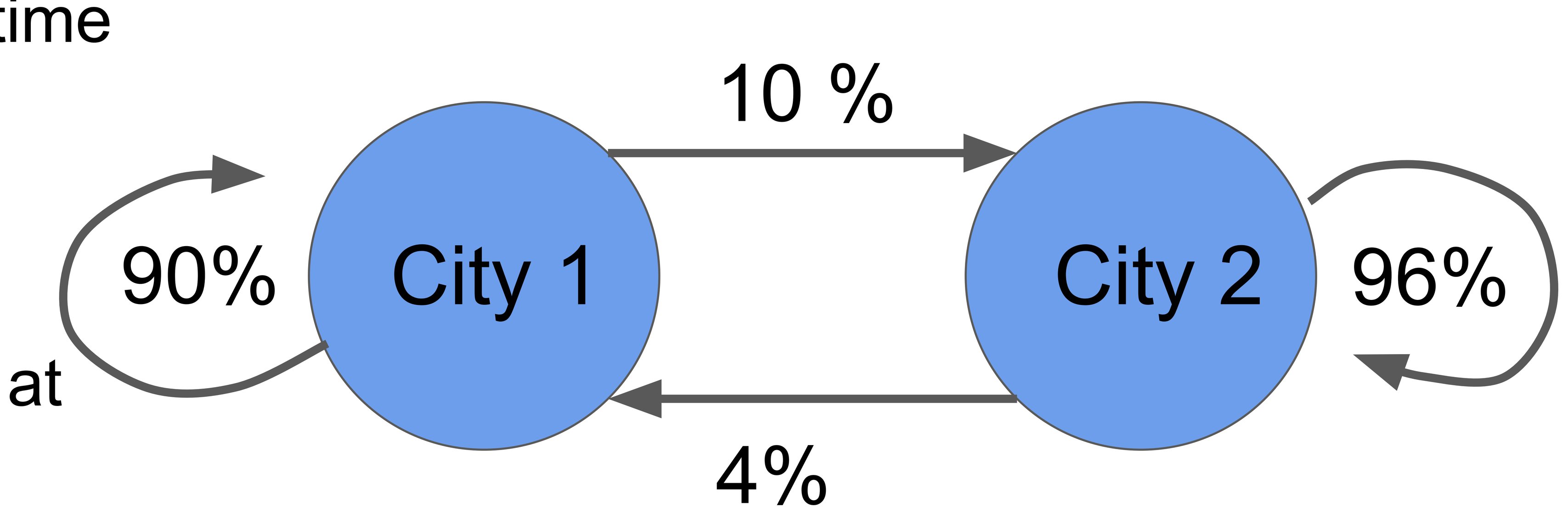
- Markov chains are a mathematical process used to model sequences of events [5]
- The probability of something in the future happening, a state change, is dependent on what happened right before it [5]
- Two types: discrete-time and continuous time Markov chains [5]
  - Discrete-time: state changes occur at specific points in time
  - Continuous-time: state changes occur at any time
- In the example to the right, there are two states, City 1 and City 2, so a state change would be moving from one city to the other or staying in current city
- Discovered by Andrei Andreyevich Markov [1]

How Markov Chains Work

- **Probability Matrix:** matrix that contains the probability of moving from one state to another [3]
- **Initial State vector:** our initial probabilities at each state [5]
- **Steady State Vector:** is a vector x such that Px=x, every Markov chain has a steady state vector. [3]
- By multiplying the state vector with the matrix it creates a new vector
- The chain is created by repeatedly multiplying the probability matrix and the new state vector.

Example

- Use Discrete-time Markov chain to determine the population of two cities after some time. Every year the population of the two cities change: 10% of City 1 moves to City 2 and 4% of City 2 moves to City 1



- Probability matrix = $\begin{bmatrix} 0.9 & 0.04 \\ 0.1 & 0.96 \end{bmatrix}$
- Initial state vector = $\begin{bmatrix} 100 \\ 100 \end{bmatrix}$

1. $\begin{bmatrix} 0.9 & 0.04 \\ 0.1 & 0.96 \end{bmatrix} \begin{bmatrix} 100 \\ 100 \end{bmatrix} = \begin{bmatrix} 94 \\ 106 \end{bmatrix}$
2. $\begin{bmatrix} 0.9 & 0.04 \\ 0.1 & 0.96 \end{bmatrix} \begin{bmatrix} 94 \\ 106 \end{bmatrix} = \begin{bmatrix} 88.84 \\ 111.16 \end{bmatrix}$
3. $\begin{bmatrix} 0.9 & 0.04 \\ 0.1 & 0.96 \end{bmatrix} \begin{bmatrix} 88.84 \\ 111.16 \end{bmatrix} = \begin{bmatrix} 84.4024 \\ 115.5976 \end{bmatrix}$

- These values are what create the Markov chain and will converge to the steady state vector
- Will lead to a steady state vector of about, $\begin{bmatrix} 57 \\ 143 \end{bmatrix}$
- Depending on the probability matrix, the rate of convergence to the steady state vector can be very fast or very slow
- From this we can see that if the change in population stays the same for both cities then population of City 1 will eventually decrease to 57, and the population of City 2 will increase to 143

Applications

- The most famous application was A.A. Markov studying the probabilities and modeling when vowels occur in the poem "Eugene Onegin" by Alexander Pushkin [1]
- More recent applications: Markov chains are commonly used in finance and to help model the stock market [2]

Further Questions

- Are Markov chains the most efficient way to model events which are dependent on what happened before?
- Are discrete-time or continuous-time Markov chains used more in real life applications?
- Could infectious diseases be modeled with continuous time Markov chain? Similar to SIR model? [4]

References

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