Fractals

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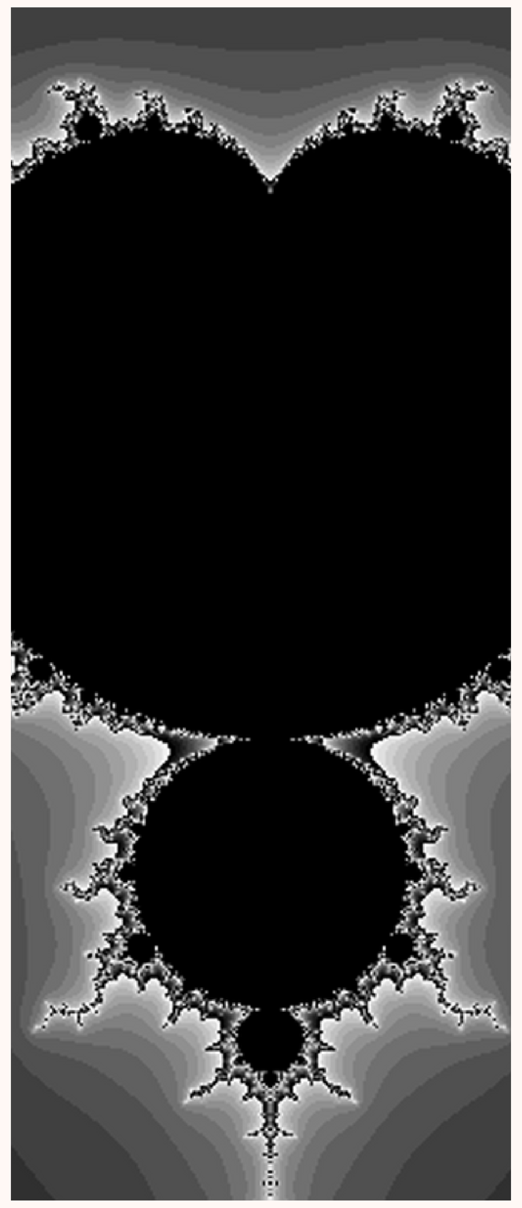
What are Fractals?

Fractals are never ending patterns that are infinitely complex, showing self-similarity on different scales. These patterns are driven by recursion, as each new component in the pattern mirrors the shape of the larger segment before it. Fractals can be found virtually everywhere from the stock market to our own bodies, and they are often used as a tool for representing infinitely complex dynamic systems that are otherwise chaotic and unpredictable.

History of Fractals

The Mandelbrot Set

- The term fractal comes from the Latin root 'fractus' which means "broken", or "fractured"
- Benoit Mandlebrot is considered the "father" of fractals, and was one of the first to model them and apply them, using them to describe the behavior of financial markets and telephone line noise.
- Mandlebrot discovered the Mandlebrot Set in 1975, which is a shape that layers several "brots" that become a single image when they converge infinitely. [5]

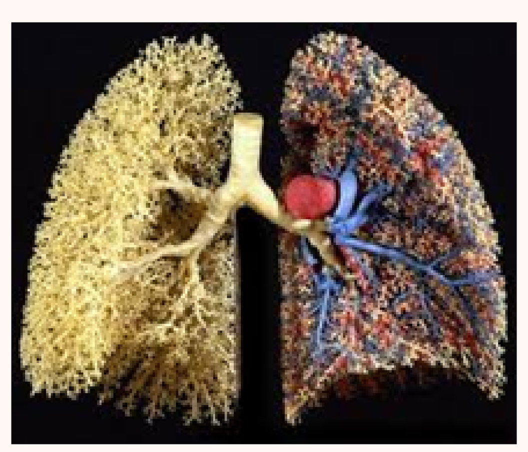


Fractals in Nature

"Clouds are not spheres, mountains are not cones, coastlines are not circles, and bark is not smooth, nor does lightning travel in a straight line"
- Mandlebrot [3]

Fractals help model the chaos of the world.

[1] many parts of this one fern leaf



[4] the branching of tracheal tubes

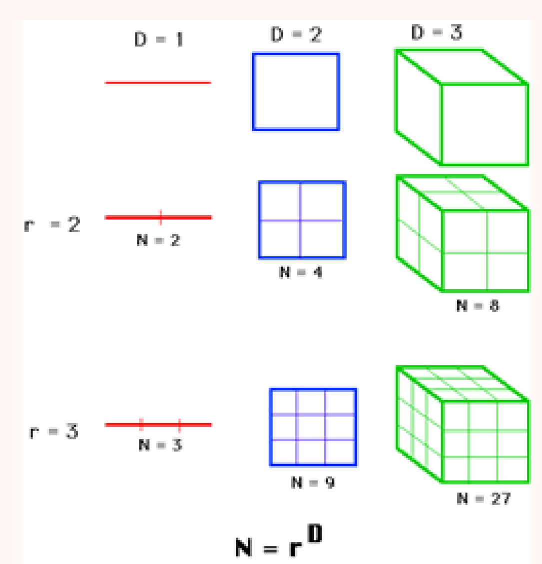
[6] tree branches growing

Self-Similarity

Fractal patterns are **self-similar** in that small segments of the fractal resemble the whole. The self similarity of fractals can either be **deterministic** or **stochastic**, meaning that the fractal pattern could follow a specific function with predictable values, or it could be randomness that is statistically self similar.

Fractal Dimension

- Measure the complexity of a fractal by quantifying how rough its surface is: the "rougher" the line, the larger the fractal dimension.
- **Dimension:** number of self similar pieces that can be created from N number of linearly independent divisions
- **Hausdorff Dimension:** $D = \log N / \log r$
  - N = number of new segments produced from each existing fractal segment, r = size of each new segment compared to the parent segment
  - Reducing size by $1/r$ in each dimension displays the number of self similar pieces that can be made
  - $N = r^D$ [7]



Fractals at Infinity

- Paradoxes are introduced when recursion is applied at infinity
  - As the fractal gets increasingly complex at smaller scales, the length of the line creating the fractal increases approaching infinity

[7] The Koch Curve above starts out with a length of 1, then a length of 4/3 on its first iteration, then a length of 16/9, and increases with each degree of complexity .

Applications

- Computers are used to model the geometric shape of fractals to study them deeper by reaching a level of detail unattainable by hand.
- Fractals are used in data management to break information down into its smallest pieces and display relationships between topics.
- Other fields using fractals include astronomy, fluid mechanics, telecommunication, surface physics, medicine, and abstract art.

This image shows a mountain river: Its shape is mirrored by all of the tiny streams that come together to form the entire river. At each scaled in level there is greater self similar detail. [3]

Further Questions

- Can fractal study be used to predict earthquake patterns and weather in general?
- In terms of big data, since there is not a single pattern being repeated over and over but in fact multiple patterns, will fractals have to be redefined?

References

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