Finding Connections Between Learning Obstacles in Introductory Proofs Courses

Sean Clark
seclark@vt.edu

Rachel Arnold
rachel.arnold@vt.edu

The Proofs Project

The main purpose of The Proofs Project is to identify and address Epistemological Obstacles (EOs) students face while taking introductory proofs courses.

What are Epistemological Obstacles (EOs)?

There are three types of obstacles a student may face when learning something new:

1. Ontogenetic obstacles - obstacles due to brain maturity
2. Didactical obstacles - obstacles due to teaching methods
3. Epistemological obstacles - obstacles when other two are not present

Important notes about EOs:

- They are necessary challenges a student faces when learning
- Often occur due to new information challenging previous ways of reasoning
- Tackling these requires a structural change in the way students approach concepts

Why Intro to Proofs Courses?

Upper level math classes focus more on proving math rather than solving calculations. These intro proof courses are then vital to students' success because they are the foundation for the advanced math courses they take in the future.

It also introduces students to logical structure, allowing them to validate their own arguments, giving them confidence and authority over their solutions and arguments in the future.

This Study

Research Questions

Through this specific study within The Proof Project, we ask:

- which EOs occur together, and how often?
- what themes exist regarding why certain EOs occur together?
- does our proposed methodology work for discovering EO co-occurrence work?

Methodology

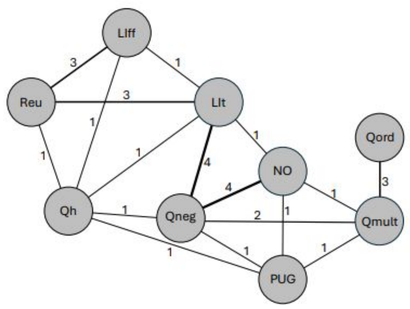
Our methodology contains 6 main steps.

1. Identify EOs through a literature review; make a codebook
2. Collect videos of students discussing topics/problems in class
3. Analyze video recordings, marking where EOs occur
4. Decide chunks where EOs would be considered to co-occur
5. Count co-occurrences per chunk to create Adjacency Matrix

Table 1. Adjacency matrix of EOs experienced in an introductory proofs course

	LIff	LIt	Qh	Qmult	Qord	Qneg	NO	PUG	Reu	Total
LIff		1	1	0	0	0	0	0	3	5
LIt	1		1	0	0	4	1	0	3	10
Qh	1	1		0	0	1	0	1	1	5
Qmult	0	0	0		3	2	1	0	0	6
Qord	0	0	0	3		0	0	0	0	3
Qneg	0	4	1	2	0		3	1	0	11
NO	0	1	0	1	0	3		1	0	6
PUG	0	0	1	0	0	1	1		0	3
Reu	3	3	1	0	0	0	0	0		7

6. Create a graph representing results



What Students Discuss

Below are examples of exercises students work on in groups that cause EOs to come into play:

Consider the statement:

There exists a positive integer x such that $x^3 - 4x^2 - x + 4 = 0$.

For each proof below, decide whether it does or does not prove this statement.

Proof 1. Suppose that $x^3 - 4x^2 - x + 4 = 0$. Then,

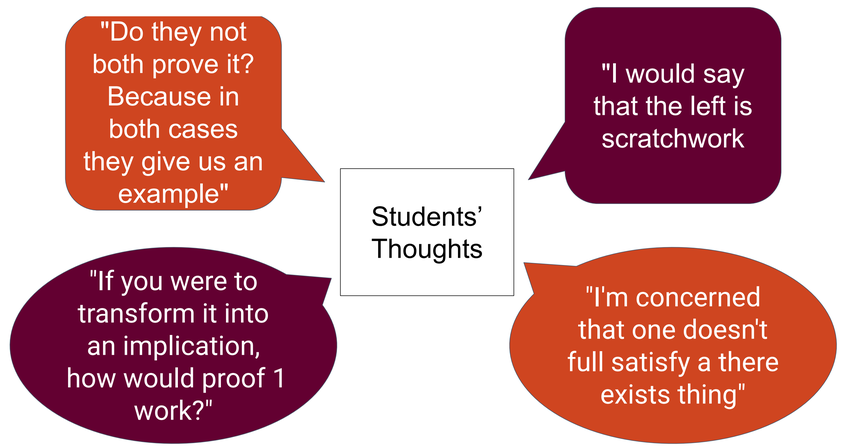
$$\begin{aligned} x^2(x-4) - (x-4) &= 0 \\ (x-4)(x^2-1) &= 0 \end{aligned}$$

So, $x = 4$ or $x = \pm 1$. Since $x = 1$ is a positive integer, the claim is true. □

Proof 2. Let $x = 1$. Then,

$$\begin{aligned} x^3 - 4x^2 - x + 4 &= (1)^3 - 4(1)^2 - (1) + 4 \\ &= 1 - 4 - 1 + 4 \\ &= 0. \end{aligned}$$

So, the claim is true. □



Experienced EOs

- Transforming Logical Implications (LIt)
- Hidden Quantification (Qh)
- Assuming Bidirectionality of Logical Implication (LIff)

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References