Modeling Disease Spread with Fractals

By: Carter Oppleman and Dean Peri

What are Fractals?

Fractal Geometry has been used to study the density of cities as well as how people move and how disease spreads. The topic gives a unique perspective and aids in making predictions in chaotic systems.

The Fractal Dimension is a metric that is used to determine how complex the fractal is. This is usually computed by following Minkowski Box-Counting method where the dimension is calculated through the following formula:

$$d_{box-counting}(X) = \frac{\ln(N(s))}{\ln\left(\frac{1}{s}\right)}$$

Where N(S) represents the number of sets of the form $[m_1 s, (m_1+1)s] \times [m_2 s, (m_2+1)s]$ where m_1, m_2 are in the set of natural numbers and s>0, which intersect s. When studying infectious disease spread, the fractal dimension is useful for determining how at-risk certain areas are for disease transmission based on spatial complexity.

Famous Fractals

Barnsley Fern - Fractal

The Barnsley Fern is a fractal that was created by Michael Barnsley and detailed in his text-book, "Fractals Everywhere". This Fractal is generated by the Iterated Function System, which is useful for generating many Fractals (i.e. Fractal interpolation).

$$f_w(x,y) = \begin{bmatrix} a & b \\ c & d \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} + \begin{bmatrix} e \\ f \end{bmatrix}$$

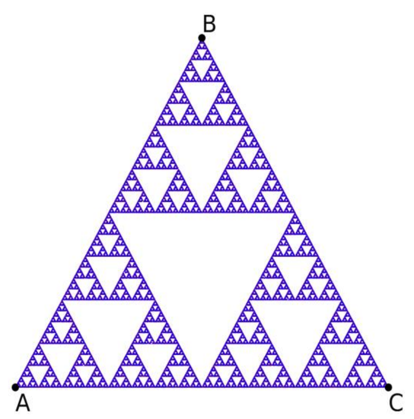
The specific transformations for this fractal are as follows:

$$f_1(x,y) = \begin{bmatrix} 0.00 & 0.00 \\ 0.00 & 0.16 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix}$$

$$f_2(x,y) = \begin{bmatrix} 0.85 & 0.04 \\ -0.04 & 0.85 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} + \begin{bmatrix} 0.00 \\ 1.60 \end{bmatrix}$$

$$f_3(x,y) = \begin{bmatrix} 0.20 & -0.26 \\ 0.23 & 0.22 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} + \begin{bmatrix} 0.00 \\ 1.60 \end{bmatrix}$$

$$f_4(x,y) = \begin{bmatrix} -0.15 & 0.28 \\ 0.26 & 0.24 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} + \begin{bmatrix} 0.00 \\ 0.44 \end{bmatrix}$$



(Note: x and y are random numbers between 0 and 1). Each transformation corresponds to a different aspect of the fractal. For example, f(1) corresponds to the stem of the Barnsley Fern. Barnsley had to determine all these coefficients through trial and error, and by studying natural ferns. Another famous Fractal is the Sierpinski triangle, which is also constructed through an IFS. The whole of the triangle is constructed using smaller replicas of the original triangle.

Our Questions

- Can fractal geometry be used to aid in modeling the spread of infectious disease cases?
- How can we calculate the fractal dimension of a city and what it represents?
- How can adding fractal geometry into infectious disease models improve predictions of spread?

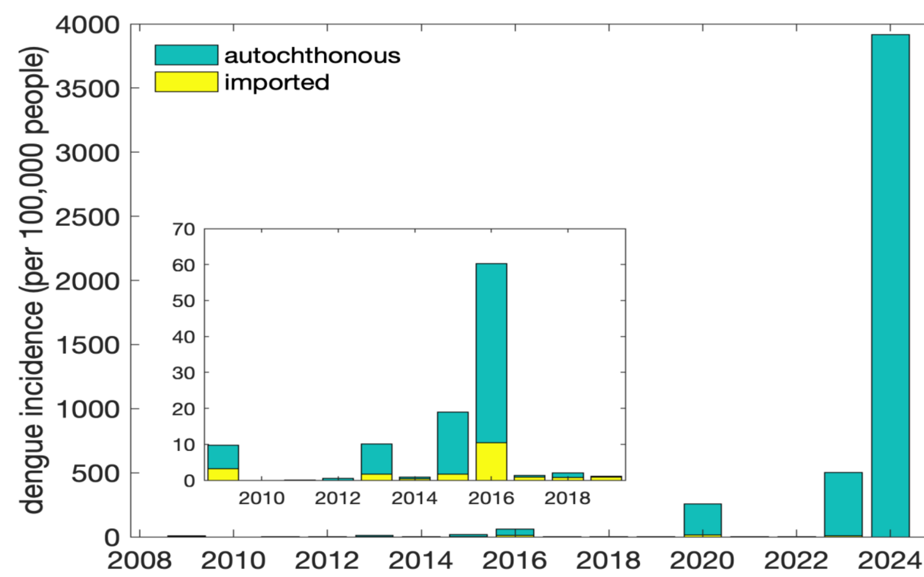
Creating Our Models

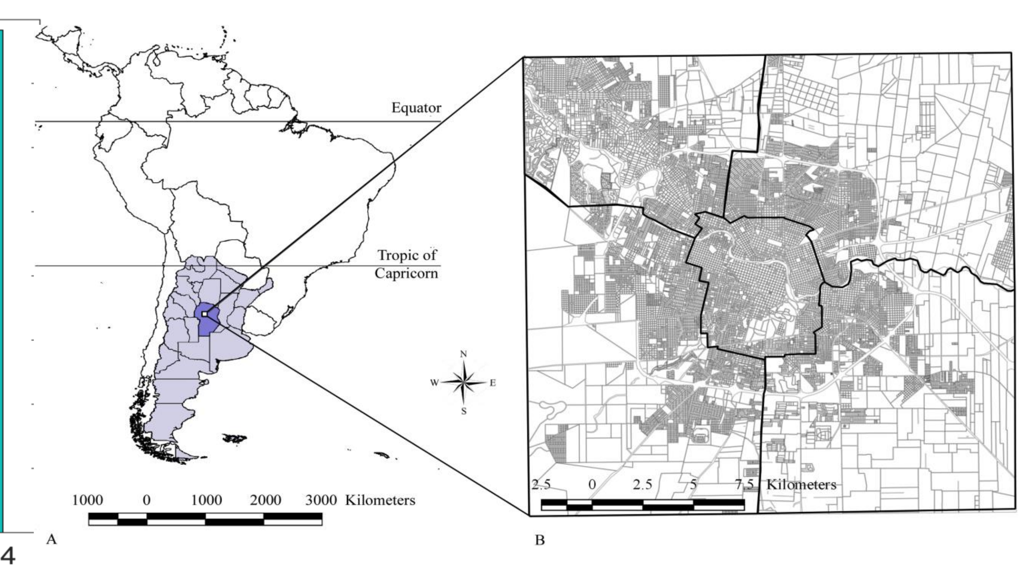
Throughout the project, we have created a total of 5 working codes we improve frequently.

- Black and white code
  - Takes a city and turns manmade places into black and natural places into white and generates the map in black and white.
- Fractal dimension calculator
  - Takes the black and white map and splits it into tiles. Once in tiles, the percentages of black or white in each tile are added together to estimate the percentage of the city that is human-made. The higher the percentage, the higher the fractal dimension.
- Fractal Interpolation
  - Approximates a given curve with a fractal. See the right column for more details.
- Final prediction map
  - We will incorporate the fractal dimension into a hotspot detection model that uses multiple variables to predict potential areas of high dengue activity throughout the city.

Case Study: Dengue in Cordoba, Argentina

- Dengue Fever is a mosquito-borne disease endemic to over 100 countries across the globe.
- Dengue is rapidly spreading into new regions, and it is estimated that almost 400 million dengue cases occur each year.
- Dengue was first reported in Cordoba, Argentina in 2009; in 2024, there were over 60,000 cases in the city.
- Cordoba is a city of about 2.1 million people in the central region of Argentina.





Future Plans

- We will calculate the Fractal dimension of more areas of interest and then conclude how at risk for disease spread those areas are. We will use the fractal dimension metric to compare disease transmission risk of different cities.
- We will also calculate the fractal dimension of the fractal interpolation graphs of the disease spread. This will give us an estimate of how complex disease spread is over time.

Fractal Interpolation

Fractal Interpolation serves as a method for modeling phenomenon that exhibit fractal properties. Disease spread is one of the instances where daily cases of the disease of interest (Dengue) exhibit jagged and irregular patterns. Thus, interpolating the data with fractals can better visually illustrate self-similarity and the patterns located within disease cases. The plot below is modeled based off the Iterated Function System of the form:

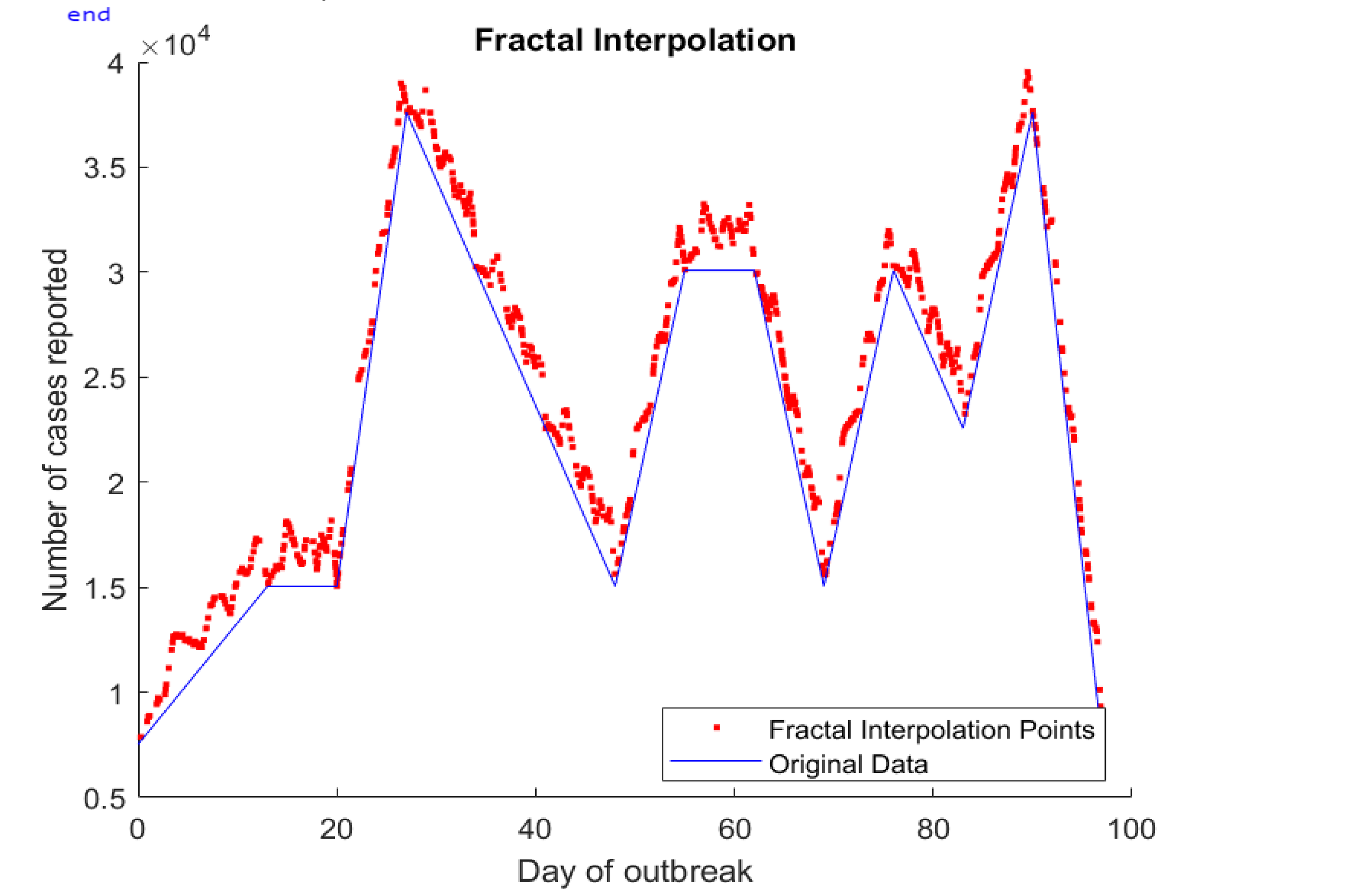
$$f\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} a_k & 0 \\ c_k & d_k \end{pmatrix} + \begin{pmatrix} b_k \\ e_k \end{pmatrix}$$

In this case x is an array where each entry represents the day of the disease out-break (Ex: x(0) = 0, day 0) and y is the array with entries corresponding to the number of cases reported that day. Each x and y entry are then run through the arrays A, C, D, B, and E, which are defined as the 'rules' of the fractal. The 'rules' are calculated as constants using formulas defined by Barnsley in "Fractals Everywhere". In the end x and y are calculated as such:

```
for n = 1:num_iter
    k = randi([1, N-1]);
    x1(n) = A(k) * x_new + B(k);
    y1(n) = C(k) * x_new + y_new * D(k) + E(k);

    %update
    x_new = x1(n);
    y_new = y1(n);

    %plot(x1(n), y1(n), 'r.'); plotted outside loop (just put this here for
    %presentation
end
```



Notice that from x = 0 to x = 20, the Fractal Interpolated function is jagged and more irregular like the graph in the latter parts, This coincides with how fractals often include one pattern that is repeated over time. (Note this is just test data).

References

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