Assessing Epistemological Obstacles Related to Euler Diagrams

Bethany Thompson
bethanyth@vt.edu

Anderson Norton
norton3@vt.edu

Purpose & Motivation

Research Question:
How can we design assessment tasks that target the epistemological obstacles students might experience in working with Euler diagrams?

Epistemological Obstacle:
Struggles related to fluidly representing logical statements and their transformations with spatial relationships within Euler diagrams.

Motivation:
I want to provide tasks and a means to assess student challenges and accomplishments related to transforming spatial and logical representations.

Background

Why do we use Euler Diagrams?
Euler diagrams are shown to better support constructing logical conclusions about an implication, as compared to Venn diagrams or symbolic forms (Mineshima et al., 2014).

One way of supporting is by populating sets with concrete examples, for which generalizations can be made into visual and symbolic forms towards fluency of content (Bronkhorst et al., 2021 and Hub & Dawkins, 2018).

Visual representation provides an external source of information for students to work with, reducing working memory load, and can be self-guiding (Mineshima et al., 2014 and Sato, Mineshima, & Takemura, 2010).

Where are Euler Diagrams used?
In the proofs course, Euler diagrams are used as an introduction to how logical implications behave and look.

Logico-Spatial Linked Structuring (Antonides, Norton, & Arnold, 2024)

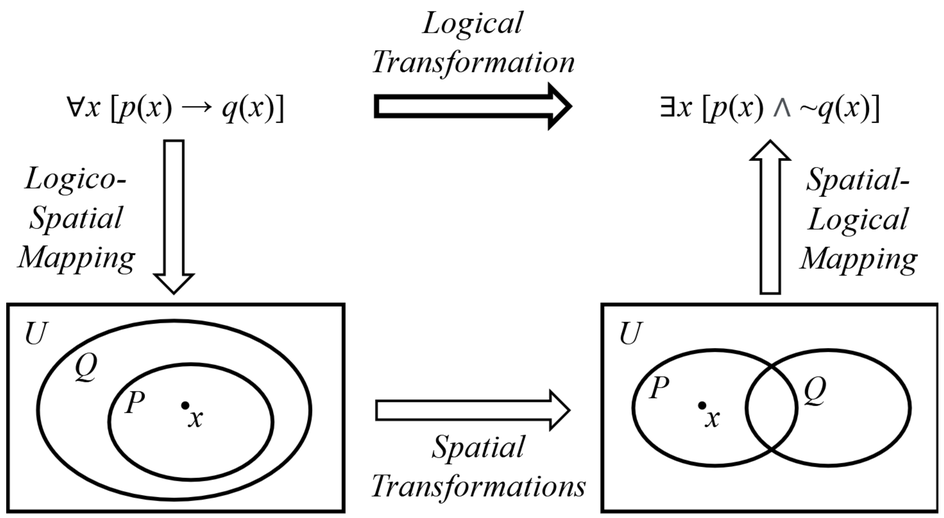

Figure 1: LSLS Framework for Spatial Transformations of Euler Diagrams

Example Task

Assume the following statement is true.
"For all quadrilaterals, if a quadrilateral is not a parallelogram, then it is not a square."

(a) Construct an Euler diagram that represents the logical implication.

(b) Now switch the labels of the circles.

(c) Write a statement/implication that describes the transformed diagram in part (b).

Reliability

How can we make sure we get consistent results on these tasks?

- Clear rubric for scoring a student's responses reduces variability in assessments across raters.
- Conduct interviews with students about their thought processes and ability to reason about these tasks.

Validity

How do we make sure these tasks assess students' logical reasoning as intended?

- Part (b) is worded in a way that does not rely on student's knowledge of definitions such as converse or negation.
- Face validity: experts and researchers provide their input on what they think the task is measuring.
- Student interpretation of what the task is asking of them (interviews).
- Content validity: aligning tasks with knowledge associated with the proofs course and implementing tasks in appropriate context.

References

Antonides, J., Norton, A., & Arnold, R. (2024). Linking Structures across Logic and Space: The Role of Euler Diagrams. *For the Learning of Mathematics*, 44(2).

Bronkhorst, H., Roorda, G., Suhre, C., & Goedhart, M. (2021). Student development in logical reasoning: Results of an intervention guiding students through different modes of visual and formal representation. *Canadian Journal of Science, Mathematics and Technology Education*, *21*(2), 378–399. https://doi.org/10.1007/s42330-021-00148-4.

Hub, A., & Dawkins, P. C. (2018). On the construction of set-based meanings for the truth of mathematical conditionals. *The Journal of Mathematical Behavior*, 50, 90-102.

Mineshima, K., Sato, Y., Takemura, R., & Okada, M. (2014). Towards explaining the cognitive efficacy of Euler diagrams in syllogistic reasoning: A relational perspective. *Journal of Visual Languages & Computing*, *25*(3), 156–169. https://doi.org/10.1016/j.jvlc.2013.08.007.

Sato, Y., Mineshima, K., & Takemura, R. (2010a). Constructing internal diagrammatic proofs from external logic diagrams. *Proceedings of the Annual Meeting of the Cognitive Science Society*, *32*, 2668–2673.

Acknowledgements

Support for this work was provided by NSF Grant No. 2141626 and by a 4-VA Collaborative Research Grant.