

Curriculum Vitae for Nicholas Loehr

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Contact Information

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Education

Ph.D., Mathematics, UC San Diego, 2003
M.A., Mathematics, UC San Diego, 2001
M.S., Computer Science, Virginia Tech, 1998
B.S., Computer Science, Virginia Tech, 1997
B.S., Mathematics, Virginia Tech, 1997

Employment

Professor of Mathematics, Virginia Tech, 2017–present.
Associate Professor of Mathematics, Virginia Tech, 2011–2017.
Associate Professor of Mathematics, United States Naval Academy, 2012–2015.
Assistant Professor of Mathematics, Virginia Tech, 2007–2011.
Assistant Professor of Mathematics, College of William and Mary, 2005–2007.
NSF Postdoctoral Research Fellow, Univ. of Pennsylvania, 2003–2005.
Adjunct Instructor, San Diego Mesa Community College, 2002.
Research Adjunct, IDA/CCR-La Jolla, 1998–present.
Teaching Assistant, UCSD Mathematics Dept., 1998–2003.
Teaching Assistant, Virginia Tech Computer Science Dept., 1997–1998.
Participant in Director’s Summer Program, Dept. of Defense, 1996, 1997.

Grants and Fellowships

- Simons Collaboration Grant for Mathematicians, \$42,000, 9/1/2019–8/31/2024.
- Simons Collaboration Grant for Mathematicians, \$35,000, 9/1/2012–8/31/2018.
- NSA Young Investigators Grant, \$30,000, 4/30/2008–4/30/2010.
- NSF Postdoctoral Research Fellowship, \$108,000, 2003–2005.
- NSF Graduate Research Fellowship, 1998–2001.

Books

1. *Bijjective Combinatorics*, CRC Press (2011), 612 pages.
2. *Advanced Linear Algebra*, CRC Press (2014), 632 pages.
3. *Combinatorics* (second edition), CRC Press (2017), 618 pages.
4. *An Introduction to Mathematical Proofs*, CRC Press (2019), 395 pages.

Refereed Journal Articles

1. (with P. Stockmeyer et al.) “Exchanging disks in the tower of Hanoi,” *Int. J. Comput. Math.* **59** (1995), 37–47.
2. “Trapezoidal lattice paths and multivariate analogues,” *Adv. in Appl. Math.* **31** (2003), 597–629.

3. “Note on André’s reflection principle,” *Discrete Math.* **280** (2004), 233–236.
4. (with L. Heath) “New algorithms for generating Conway polynomials over finite fields,” *J. Symbolic Comput.* **38** (2004), 1003–1024.
 - 4a. (with L. Heath) “New algorithms for generating Conway polynomials over finite fields,” *Proceedings of the Tenth Annual ACM-SIAM Symposium on Discrete Algorithms (SODA)*, 429–437 (1999). Conference version of journal article.
5. (with J. Remmel) “Conjectured combinatorial models for the Hilbert series of generalized diagonal harmonics modules,” *Electron. J. Combin.* **11** (2004) research paper R68; 64 pages (electronic).
6. (with G. Warrington and H. Wilf) “The combinatorics of a three-line circulant determinant,” *Israel J. Math.* **143** (2004), 141–156.
7. “Permutation statistics and the q, t -Catalan sequence,” *European J. Combin.* **26** (2005), 83–93.
8. “Combinatorics of q, t -parking functions,” *Adv. in Appl. Math.* **34** (2005), 408–425.
9. (with J. Haglund, M. Haiman, J. Remmel, and A. Ulyanov) “A combinatorial formula for the character of the diagonal coinvariants,” *Duke Math. J.* **126** (2005), 195–232.
10. “Conjectured statistics for the higher q, t -Catalan sequences,” *Electron. J. Combin.* **12** (2005) research paper R9; 54 pages (electronic).
11. (with J. Haglund and M. Haiman) “Combinatorial theory of Macdonald polynomials I: Proof of Haglund’s formula,” *Proc. Natl. Acad. Sci. USA* **102** #8 (2005), 2690–2696+cover illustration.
12. (with J. Haglund and J. Remmel) “Statistics on wreath products, perfect matchings, and signed words,” *European J. Combin.* **26** (2005), 835–868.
13. (with J. Haglund and M. Haiman) “A combinatorial formula for Macdonald polynomials,” *J. Amer. Math. Soc.* **18** (2005), 735–761.
14. (with J. Haglund) “A conjectured combinatorial formula for the Hilbert series for diagonal harmonics,” *Discrete Math.* **298** (2005), 189–204.
15. (with A. Mendes) “Bijective matrix algebra,” *Linear Algebra Appl.* **416** (2006), 917–944.
16. (with M. Can) “A proof of the q, t -square conjecture,” *J. Combin. Theory Ser. A* **113** (2006), 1419–1434.
 - 16a. (with M. Can) “A proof of the q, t -square conjecture,” *Proceedings of FPSAC* (2006), 62–70. Conference version of journal article.
17. (with G. Warrington) “Square q, t -lattice paths and $\nabla(p_n)$,” *Trans. Amer. Math. Soc.* **359** (2007), 649–669.
18. “The major index specialization of the q, t -Catalan,” *Ars Combin.* **83** (2007), 145–160.
19. “A direct proof that row rank equals column rank,” *College Math. J.* **38** (2007), 300–301.

20. (with G. Warrington) “Nested quantum Dyck paths and $\nabla(s_\lambda)$,” *Intl. Math. Research Notices* **2008** (2008), article ID rnm157, 29 pages.
21. (with J. Haglund and M. Haiman) “A combinatorial formula for non-symmetric Macdonald polynomials,” *Amer. J. Math.* **130** (2008), 359–383.
22. (with K. Benedetto) “Tiling problems, automata, and tiling graphs,” *Theoret. Comput. Sci.* **407** (2008), 400–411.
23. (with B. Sagan and G. Warrington) “A human proof for a generalization of Shalosh B. Ekhad’s 10^n Lattice Paths Theorem,” *Ars Combin.* **89** (2008), 421–429.
24. (with G. Warrington) “A continuous family of partition statistics equidistributed with length,” *J. Combin. Theory Ser. A* **116** (2009), 379–403.
25. (with J. Remmel), “Rook-by-rook rook theory: bijective proofs of rook and hit equivalences,” *Adv. Appl. Math.* **42** (2009), 483–503.
26. (with E. Brown) “Why is $\text{PSL}(2, 7) \cong \text{GL}(3, 2)$?” *Amer. Math. Monthly* **116** (2009), 727–732.
27. (with I. Gessel) “Note on enumeration of partitions contained in a given shape,” *Linear Algebra Appl.* **432** (2010), 583–585.
28. “Abacus proofs of Schur function identities,” *SIAM J. Discrete Math.* **24** (2010), 1356–1370.
29. (with C. Savage) “Generalizing the combinatorics of binomial coefficients via ℓ -nomials,” *Integers* **10** (2010), paper A45, 531–558.
30. (with E. Egge and G. Warrington) “From quasisymmetric expansions to Schur expansions via a modified inverse Kostka matrix,” *European J. Combin.* **31** (2010), 2014–2027.
31. (with J. Remmel) “A computational and combinatorial exposé of plethystic calculus,” *J. Algebraic Combin.* **33** (2011), 163–198.
32. (with E. Niese) “Recursions and divisibility properties for combinatorial Macdonald polynomials,” *Discrete Math. Theor. Comput. Sci.* **13** (2011), 21–42.
33. (with G. Warrington) “Quasisymmetric expansions of Schur-function plethysms,” *Proc. Amer. Math. Soc.* **140** (2012), 1159–1171.
34. (with W. Kaczynski, L. Leemis, and J. McQueston) “Nonparametric random variate generation using a piecewise-linear cumulative distribution function,” *Comm. Statist. Simulation Comput.* **41** (2012), 449–468.
35. (with W. Kaczynski, L. Leemis, and J. McQueston) “Bivariate nonparametric random variate generation using a piecewise-linear cumulative distribution function,” *Comm. Statist. Simulation Comput.* **41** (2012), 469–496.
36. (with E. Niese) “A bijective proof of a factorization formula for specialized Macdonald polynomials,” *Ann. Comb.* **16** (2012), 815–828.
37. (with R. D. Mauldin) “Bijective proofs of Jensen’s and Mohanty–Handa’s identities,” *Appl. Anal. Discrete Math.* **7** (2013), 11–24.
38. (with K. Benedetto) “Domino tiling graphs,” *Ars Combin.* **109** (2013), 3–29.

39. (with K. Lee and L. Li) “Limits of modified higher q, t -Catalan numbers,” *Electron. J. Combin.* **20(3)** (2013), research paper P4, 23 pages (electronic).
40. (with L. Serrano and G. Warrington) “Transition matrices for symmetric and quasisymmetric Hall–Littlewood polynomials,” *J. Combin. Theory Ser. A* **120** (2013), 1996–2019.
 - 40a. (with L. Serrano and G. Warrington) “Transition matrices for symmetric and quasisymmetric Hall–Littlewood polynomials,” *Discrete Math. Theor. Comput. Sci. proc.* **AS** (2013), 301–312. Conference version of journal article, presented at FPSAC 2013.
41. (with E. Green and A. Pelley) “ C -rings, coproducts, and reflection functors,” *J. Algebra Appl.* **13** (2014), paper 1350071 (22 pages).
42. “Classifying nilpotent maps via partition diagrams,” *College Math. J.* **45** (2014), 108–115.
43. (with K. Barrese, J. Remmel, and B. Sagan) “ m -level rook placements,” *J. Combin. Theory Ser. A* **124** (2014), 130–165.
44. (with E. Niese) “New combinatorial formulations of the shuffle conjecture,” *Adv. Appl. Math.* **55** (2014), 22–47.
45. (with K. Lee and L. Li) “Combinatorics of certain higher q, t -Catalan polynomials: chains, joint symmetry, and the Garsia-Haiman formula,” *J. Algebraic Combin.* **39** (2014), 749–781.
46. (with D. Armstrong and G. Warrington) “Sweep maps: a continuous family of sorting algorithms,” *Adv. Math.* **284** (2015), 159–185.
 - 46a. (with G. Warrington) “Sweep maps for lattice paths,” *Discrete Math. Theor. Comput. Sci. proc.* **AT** (2014), 667–678. Conference version of journal article, presented as a poster at FPSAC 2014.
47. (with D. Armstrong and G. Warrington) “Rational parking functions and Catalan numbers,” *Ann. Comb.* **20** (2016), 21–58.
48. (with K. Barrese, J. Remmel, and B. Sagan) “Bijections on m -level rook placements,” *European J. Combin.* **57** (2016), 13–35.
49. (with A. Wills) “Abacus-tournament models for Hall–Littlewood polynomials,” *Discrete Math.* **339** (2016), 2423–2445.
50. “Variants of the RSK algorithm adapted to combinatorial Macdonald polynomials,” *J. Combin. Theory Ser. A* **146** (2017), 129–164.
51. (with K. Lee and L. Li) “A combinatorial approach to the symmetry of q, t -Catalan numbers,” *SIAM J. Discrete Math.* **32** (2018), 191–232.
52. (with T. S. Michael) “The combinatorics of evenly spaced binomial coefficients,” *Integers* **18** (2018), paper A89 (20 pages).
53. (with G. Warrington) “Quasisymmetric and Schur expansions of cycle index polynomials,” *Discrete Math.* **342** (2019), 113–127.
54. “Successor algorithms via counting arguments,” *Bull. Inst. Combin. Appl.* **86** (2019), 101–122.
55. “Abacus proofs of Cauchy Product Identities for Schur polynomials,” *Ann. Comb.* **23** (2019), 367–389.

- Preprints** 56. (with G. Warrington) “Abacus-histories and the combinatorics of creation operators,” in preparation.

Other Publications

- *Multivariate analogues of Catalan numbers, parking functions, and their extensions*. Doctoral dissertation, University of California at San Diego (2003), 267 pages. Advisor: Professor Jeffrey B. Remmel (1948—2017).
- Foreword to part 1 of the special issue in memory of Jeff Remmel, *J. Comb.* **10** issue 3 (2019), 409–410.
- Foreword to part 2 of the special issue in memory of Jeff Remmel, *J. Comb.* **10** issue 4 (2019), 596–598.

Invited Talks, Conferences, and Workshops

1. “Conjectured statistics for the higher q, t -Catalan polynomials,” Combinatorics seminar, UC San Diego, 4/2002.
2. “Conjectured statistics for the higher q, t -Catalan polynomials,” Mini-symposium on diagonal harmonics and Macdonald polynomials, SIAM Conference on Discrete Mathematics, San Diego, CA, 8/2002.
3. “Diagonal harmonics, Catalan numbers, and parking functions,” invited hour-long colloquium, University of Louisville, 2/2003.
4. “Lattice paths and Catalan numbers,” invited colloquium, Santa Clara University, 2/2003.
5. “Diagonal harmonics, Catalan numbers, and parking functions,” invited hour-long colloquium, Wake Forest University, 2/2003.
6. Combinatorics seminars (series of 4), Univ. of Pennsylvania, 2003–2004.
7. “The major index specialization of the q, t -Catalan,” Mini-symposium on Macdonald polynomials and diagonal harmonics, SIAM Conference on Discrete Mathematics, Nashville, TN, 6/2004.
8. “Combinatorial Macdonald polynomials,” invited hour-long colloquium, College of William and Mary, 1/2005.
9. “Combinatorial Macdonald polynomials,” invited hour-long colloquium, U.S. Naval Academy, 2/2005.
10. “Parking functions, trees, and diagonal harmonics,” invited hour-long colloquium, University of North Texas, 2/2005.
11. “Parking functions, trees, and diagonal harmonics,” invited hour-long seminar, University of Delaware, 4/2005.
12. “Introduction to Macdonald polynomials,” AIM workshop on Generalized Kostka Polynomials, Palo Alto, CA, 7/2005.
13. “Parking functions, trees, and diagonal harmonics,” invited hour-long colloquium, College of William and Mary, 10/2005.
14. “The q, t -Catalan theorem and the q, t -square theorem,” invited 25-minute talk, AMS/MAA Joint Meetings, San Antonio, TX, 1/2006.

15. "The Exogenesis of the `hans.math.upenn.edu`-Warrington-Loehr 10^n Conjecture," contributed talk, 37th Southeastern International Conference on Combinatorics, Graph Theory, and Computing, 3/2006.
16. "Building Macdonald polynomials," invited hour-long combinatorics seminar, NC State University, 4/2006.
17. "A brief history of the 10^n problem," invited hour-long combinatorics seminar, The Ohio State University, 5/2006.
18. "The q, t -Catalan theorem and the q, t -square theorem," invited hour-long seminar, University of South Carolina, 12/2006.
19. "Parking functions, trees, and diagonal harmonics," invited hour-long colloquium, Virginia Tech, 12/2006.
20. "Bijective matrix inversion," invited 25-minute talk, AMS Section Meeting, Davidson, NC, 3/2007.
21. "Combinatorial aspects of the Bergeron-Garsia nabla operator," international invited hour-long talk, Workshop on Combinatorial Hopf Algebras and Macdonald Polynomials, CRM, Montreal, Canada, 5/2007.
22. "Symmetric and non-symmetric Macdonald polynomials," international invited hour-long talk, Workshop on Applications of Macdonald Polynomials, BIRS, Banff, Canada, 9/2007.
23. "Quantum lattice paths and bijective subtraction," invited 25-minute talk, AMS Section Meeting, Murfreesboro, TN, 11/2007.
24. "Combinatorial aspects of the Bergeron-Garsia nabla operator," international invited hour-long talk, Fields Institute, Toronto, 1/2008.
25. "Bijections, lattice paths, and probability," invited hour-long colloquium talk, University of Houston, 9/2008.
26. "Tiling bijections via finite automata," contributed talk, AMS/MAA joint meetings, Washington D.C., 1/2009.
27. "Rook-by-rook rook theory," invited hour-long colloquium talk, California Polytechnic University, 2/2009.
28. "Parking functions, trees, and diagonal harmonics," invited hour-long colloquium talk, Tulane University, 3/2009.
29. "Rook-by-rook rook theory," invited 25-minute talk, AMS Section Meeting, NC State University, 4/2009.
30. "A continuum of partition statistics," invited hour-long seminar, NC State University, 5/2009.
31. "The q, t -Catalan theorem and the q, t -square theorem," invited hour-long colloquium talk, University of Virginia, 10/2009.
32. "A continuum of partition statistics," invited 25-minute talk, AMS/MAA Joint Meetings, San Francisco, CA, 1/2010.
33. "The q, t -Catalan theorem and the q, t -square theorem," hour-long colloquium talk, Virginia Tech, 3/2010.
34. "A modified inverse Kostka matrix," invited 25-minute talk, AMS Section Meeting, St. Paul, MN, 4/2010.

35. “Macdonald polynomials in representation theory and combinatorics,” hour-long colloquium talk, University of Miami, 10/2010.
36. “Rook-by-rook rook theory,” invited hour-long colloquium talk, The College of William and Mary, 4/2011.
37. “Parking functions, trees, and diagonal harmonics,” invited hour-long algebra seminar talk, Wayne State University, 10/2011.
38. “Parking functions, trees, and diagonal harmonics,” invited hour-long colloquium talk, United States Naval Academy, 2/2012.
39. “A bijective proof of a factorization formula for specialized Macdonald polynomials,” invited 25-minute talk, AMS Section Meeting, Tampa, FL, 3/2012.
40. “Rook-by-rook rook theory,” invited 25-minute talk, AMS Section Meeting, New Orleans, LA, 10/2012.
41. “Bijective proofs of Jensen’s identity and Mohanty–Handa’s identity,” invited 25-minute talk, AMS Section Meeting, Ames, IA, 4/2013.
42. “Permutation statistics and q, t -Catalan numbers,” USNA Mathematics Department Basic Notions seminar, 10/2013.
43. “Sweep maps and generalized q, t -Catalan numbers,” invited 25-minute talk, AMS Section Meeting, Halifax, Nova Scotia, 10/2014.
44. “Binomial coefficients, rational Catalan numbers, and their q -analogues,” invited 25-minute talk, AMS/MAA Joint Meetings, San Antonio, TX, 1/2015.
45. “Rook theory 101,” invited hour-long talk, Marshall University Active Research Initiative (ARI) lecture series, Huntington, WV, 9/2015.
46. “Sweep maps and bounce paths,” invited hour-long talk, Marshall University Active Research Initiative (ARI) lecture series, Huntington, WV, 9/2015.
47. “Variants of the RSK algorithm adapted to combinatorial Macdonald polynomials,” invited 25-minute talk, AMS Spring Southeastern Sectional Meeting, Athens, GA, 3/2016.
48. “Algebraic and combinatorial Macdonald polynomials,” invited hour-long colloquium, Rocky Mountain Algebraic Combinatorics Seminar, Colorado State University, Fort Collins, CO, 4/2016.
49. “RSK algorithms and combinatorial Macdonald polynomials,” invited hour-long plenary address, CombinaTexas 2016 Conference, Texas A&M University, College Station, TX, 5/2016.
50. “Chain decompositions for q, t -Catalan numbers,” contributed talk, AMS/MAA Joint Mathematics Meetings, Atlanta, GA, 1/2017.
51. “Algebraic and combinatorial Macdonald polynomials,” invited hour-long colloquium talk, Virginia Tech Mathematics Department, Blacksburg, VA, 2/2017.
52. “Rook theory 101,” invited hour-long colloquium talk, St. Mary’s College of Maryland, St. Mary’s City, MD, 3/2018.
53. “Chain decompositions for q, t -Catalan numbers,” invited 25-minute talk, AMS Spring Southeastern Sectional Meeting, Vanderbilt University, Nashville, TN, 4/2018.

54. “The combinatorics of evenly spaced binomial coefficients,” invited 25-minute talk, AMS/MAA Joint Mathematics Meetings, Baltimore, MD, 1/2019.
55. “Chain decompositions for q, t -Catalan numbers,” invited 25-minute talk, AMS/MAA Joint Mathematics Meetings, Baltimore, MD, 1/2019.

Honors and Awards

- Virginia Tech MAA Student Chapter Math Professor of the Year, 2018.
- Four Virginia Tech “Thank a Teacher” recognitions, 2017, 2019.
- Virginia Tech Favorite Faculty award, Spring 2016.
- Virginia Tech Math Club Professor of the Year, 2010.
- Virginia Tech Math Club Professor of the Year, 2008.
- Simon Teaching Award, College of William and Mary Mathematics Dept., 2007.
- Good Teaching Award, U. Penn. Mathematics Dept., Spring 2004.
- NSF Postdoctoral Research Fellowship, 2003–2005.
- NSF Graduate Research Fellowship, 1998–2001.
- Two-time Virginia Tech regional math contest winner, 1994 and 1996.
- Silver medal recipient, 26th Intl. Chemistry Olympiad, Oslo, Norway, 1994.
- Virginia Tech Presidential Scholar, 1994–1997.
- Valedictorian, Midlothian High School, 1994.

Advisees

- Katherine Benedetto, William and Mary, senior thesis on tilings.
- Elizabeth Niese, Virginia Tech, doctoral thesis on Macdonald polynomials.
- Andy Wills, Virginia Tech, doctoral thesis on abacus-tournament models for Hall–Littlewood polynomials.

Professional Service

- Co-organizer, Special Session on Combinatorial Enumeration, Optimization, Geometry, and Statistics, AMS Fall 2007 Southeastern Section Meeting, Murfreesboro, TN, Nov. 3–4, 2007.
- Co-organizer, Special Session on Symmetric Functions, Quasisymmetric Functions, and the Associated Combinatorics, AMS Spring 2012 Eastern Section Meeting, Washington, D.C., March 17–18, 2012.
- Program Committee Member, Formal Power Series and Algebraic Combinatorics Conference (FPSAC), 2015.
- Project NExT Consultant, 2015.
- Guest Editor for Special Issues of *Journal of Combinatorics* (Vol. 10 issues 3 and 4) in memory of Professor Jeffrey Remmel, 2017–2019.

Courses Taught

at the United States Naval Academy:

- SM221 (Calculus 3 with Vector Fields)
- SM221S (Honors Calculus 3)
- SM239 (Probability and Statistics)
- SM242 (Discrete Mathematics and Probability)

- SM261 (Matrix Theory)
- SM291 (Fundamentals of Mathematics)
- SM411 (Introduction to Complex Analysis)

at Virginia Tech:

- Math 1114H (Honors Linear Algebra)
- Math 2204 (Multivariable Calculus)
- Math 2204H (Honors Multivariable Calculus)
- Math 3034 (Proofs)
- Math 3124 (Modern Algebra)
- Math 3214 (Vector Calculus)
- Math 3224 (Advanced Calculus)
- Math 4124 (Abstract Algebra)
- Math 4175 (Cryptography 1)
- Math 4176 (Cryptography 2)
- Math 4225 (Real Analysis 1)
- Math 5114 (Topics in Algebra)

at the College of William and Mary:

- Math 112 (Integral Calculus)
- Math 212 (Multivariable Calculus)
- Math 214 (Foundations of Mathematics)
- Math 401 (Probability)
- Math 430 (Abstract Algebra 2)
- Math 432 (Combinatorics)

Refereeing

- NSA Mathematical Sciences Grant Program
- NSERC Discovery Grants (Canada)
- START Programme (FWF Austrian Science Fund)
- Chapman and Hall/CRC Press (book reviews)
- Cambridge University Press (book chapter review)
- American Mathematical Society (book review)
- Journal of the American Mathematical Society
- Advances in Mathematics
- Duke Mathematical Journal
- Transactions of the American Mathematical Society
- Journal of Combinatorial Theory, Series A
- Applied Mathematics and Computation

- Selecta Mathematica
- Journal of Algebraic Combinatorics
- Linear Algebra and its Applications
- Reports on Mathematical Physics
- European Journal of Combinatorics
- International Math Research Notices
- Annals of Combinatorics
- Combinatorica
- SIAM Journal on Discrete Mathematics
- Proceedings of the American Mathematical Society
- Electronic Journal of Combinatorics
- Information Sciences
- Advances in Applied Mathematics
- The Ramanujan Journal
- Discrete Mathematics
 - nominated by the Editorial Board as a **most valued referee**, 2014.
- FPSAC Conference Proceedings, currently published in the journal Discrete Mathematics and Theoretical Computer Science
- Order
- Central European Journal of Mathematics
- American Mathematical Monthly
- Integers
- Australasian Journal of Combinatorics
- Fibonacci Quarterly
- Journal of Integer Sequences
- Applicable Analysis and Discrete Mathematics
- Operators and Matrices
- Journal of Combinatorics
- Séminaire Lotharingien de Combinatoire
- Bulletin of the ICA
- Ars Combinatoria
- International Journal of Mathematics and Mathematical Sciences
- Electronic Antiquity